Noise Field of a Propeller with Angular Inflow

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The propeller blade forces are replaced by doublet and quadrupole distributions for the acceleration potential. An essential step in the solution is the derivation of a near-field and a far-field formula for the pressure due to a point force in arbitrary motion within a uniformly moving acoustic medium. The discrete frequency spectrum and the three-dimensional directionality of the noise radiated from a propeller with angular inflow are obtained by a conventional Fourier series expansion of the signature. The final analytical formulas can be evaluated with simple calculation.

Introduction

MANY papers on the noise of propellers and helicopter rotors have been published in the last few years. A review may be found in a paper by Farassat.[1] Goldstein[2] has presented a fundamental equation for the sound generation in the presence of solid boundaries. This equation includes the Ffowcs-Williams-Hawkings equation[3] for rigid surfaces and describes the sound in terms of a stationary coordinate system. Therefore, the pressure signal is given in terms of the source position at the time of its generation. However, the present paper, similar to early publications,[4-6] starts with a source in a uniformly moving medium. The additional effects of rotation are dealt with by a Dirac delta function (see Morfey[7]).

A monopole of the acceleration potential in a uniformly moving medium is taken first. The effect of forward speed is accounted for by a Lorentz-Galilean transformation. Pressure doublets and pressure quadrupoles are derived from the monopole solution in such a way that their directions with respect to that of the uniform flow can be chosen arbitrarily. Equations (16) and (17) describing the near- and far-field pressure in vector notation are the basic results of this paper. The remaining work to be done is simply to perform a vector multiplication and a conventional Fourier series expansion. The paper is restricted to the effect of loading, which for a propeller with angular inflow very often is the most important origin of noise. Asymmetric disk loading[8] is included. At transonic blade-tip speeds, thickness noise exceeds loading noise,[9] and this may be described by a velocity potential.[6] However, analysis with a velocity potential is different from that with an acceleration source, especially for the so-called quadrupole.[10] The resulting formulas for the loading noise due to a propeller with angular inflow are so simple that they can be evaluated easily with small calculators during experiments.

Acceleration Source in Arbitrary Motion within a Moving Medium

The fundamental solution for a small pressure perturbation p due to an oscillating spherical acceleration source held fixed in a uniformly moving unbounded fluid is given by

$$p = \frac{-Q(\tau)}{4\pi r} \tag{1}$$

where $Q(\tau)$ is the source strength evaluated at the retarded time

$$\tau = t - r_{ph}/a \tag{2}$$

Variable t is the time of observation in a source-fixed coordinate system, a the speed of sound, and r_{ph} the phase radius

$$r_{ph} = (M_\infty x + r)/\beta^2 \tag{3}$$

with $M_\infty = U_\infty/a$ as Mach number of uniform flow, $\beta^2 = 1 - M_\infty^2$, and r from

$$r = x + \beta^2(y^2 + z^2) \tag{4}$$

The directionality of such a pressure monopole is illustrated in Fig. 1. The solution of Eq. (1) may be viewed either as a solution of the wave equation for the pressure in a uniformly moving medium or as a Lorentz-Galilean transformation[11] to the corresponding solution for a still medium.

A source in arbitrary motion within a uniformly moving fluid may be represented likewise by an infinite distribution of the mentioned fixed-pressure monopoles having a nonzero source strength at the instant at which the moving source is at the position $\boldsymbol{\eta}$ of the fixed sources. The three-dimensional Dirac δ function is introduced to replace Q by $Q(\tau)\delta$. The new $Q(\tau)$ is a function of retarded time only. The sound field due to this distribution of fundamental solutions is thus given by

$$p = \frac{1}{4\eta}\int_{-\infty}^{+\infty}\left[\frac{Q(\tau)}{r}\right][\delta(\boldsymbol{\eta} - \boldsymbol{\eta}_o)]\,\mathrm{d}V(\boldsymbol{\eta}) \tag{5}$$

The quantities inside the square brackets have to be evaluated at the retarded time. In Eq. (5), $\delta = \delta(\boldsymbol{\eta} - \boldsymbol{\eta}_o)$ such that $\boldsymbol{\eta}_o$ is the position of existence of the function. Variable $\boldsymbol{\eta}_o$ is a function of retarded time. To evaluate the integral of $|\delta|$, the variable is changed from $\boldsymbol{\eta}$ to $(\boldsymbol{\eta} - \boldsymbol{\eta}_o)$. Now applying the chain rule to the derivation of $(\boldsymbol{\eta} - \boldsymbol{\eta}_o)$ with respect to

$$\nabla_{\boldsymbol{\eta}}\left(\nabla_{\boldsymbol{\eta}} = \left\{\frac{\partial}{\partial\eta_x}, \frac{\partial}{\partial\eta_y}, \frac{\partial}{\partial\eta_z}\right\}\right)$$

we have

$$\nabla_{\boldsymbol{\eta}} \cdot (\boldsymbol{\eta} - \boldsymbol{\eta}_o) = 1 - \nabla_{\boldsymbol{\eta}}\tau \cdot \frac{\mathrm{d}\boldsymbol{\eta}_o}{\mathrm{d}\tau} \tag{6}$$

where

$$\frac{\mathrm{d}\boldsymbol{\eta}_o}{\mathrm{d}\tau} = a\boldsymbol{M} \tag{7}$$

is the instantaneous velocity of the source superimposed on the velocity $a\boldsymbol{M}_\infty$ of the uniform flow. Variable $\boldsymbol{M}$ is the source convection Mach number vector. With the retarded time $\tau = t - r_{ph}(\boldsymbol{x} - \boldsymbol{\eta})/a$ where $\boldsymbol{x}(x,y,z)$ is the observer position, we have

$$\nabla_{\boldsymbol{\eta}}\tau = \nabla r_{ph}/a \tag{8}$$

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where $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ such that

$$dV(\boldsymbol{\eta} - \boldsymbol{\eta}_o) = (1 - \nabla r_{ph} \cdot \boldsymbol{M})\, dV(\boldsymbol{\eta}) \tag{9}$$

Equation (5) may now be integrated directly into $(\boldsymbol{\eta} = \boldsymbol{\eta}_o)$ giving

$$p = \frac{Q(\tau)}{4r(1 - \nabla' r_{ph} \cdot \boldsymbol{M})\pi} \tag{10}$$

where ∇' means that for this derivation of $r_{ph} = r_{ph}[\boldsymbol{x} - \boldsymbol{\eta}_o(\tau)]$, the retarded time τ is independent of $\boldsymbol{x}$. The component of the instantaneous source convection Mach number vector in the direction of the phase gradient is $\nabla r_{ph} \cdot \boldsymbol{M}$, with $r_{ph} = r_{ph}(\boldsymbol{x} - \boldsymbol{\eta}_o)$ and $r(\boldsymbol{x} - \boldsymbol{\eta}_o)$ defined corresponding to Eqs. (3) and (4), respectively.

Pressure monopoles have never been used to represent a sound source. However, the use of pressure doublets is quite common in descriptions of the effects of aerodynamic forces.[6] Then, if the blade force $\boldsymbol{F}$ is given by pressure difference on an infinitely thin blade, the acceleration potential is given by a distribution of pressure doublets. Since the propeller must give a certain thrust, it follows that the pressure will be negative on the front side and positive on the rear side of the blade. The relation between pressure and acceleration potential ψ is $p = -\rho\psi$, where ρ is the density. Therefore, a source giving a negative value for ψ with source strength $-Q$ is assumed to be on the rear side at position $\boldsymbol{\eta}_o$ and a sink with an equal but opposite strength $+Q$ on the front side at position $\boldsymbol{\eta}_o + \S$, where $\S$ is the finite distance vector from the source to the sink. Then

$$\psi_{\text{sink}}(\boldsymbol{x} - \boldsymbol{\eta} - \S, \tau) = \psi_{\text{sink}}(\boldsymbol{x} - \boldsymbol{\eta}, \tau) - \nabla \cdot \S\psi_{\text{sink}}(\boldsymbol{x} - \boldsymbol{\eta}, \tau)$$

Hence,

$$\psi_{\text{source}}(\boldsymbol{x} - \boldsymbol{\eta}, \tau) + \psi_{\text{sink}}(\boldsymbol{x} - \boldsymbol{\eta}, \tau) = \nabla\, \S\psi_{\text{sink}}(\boldsymbol{x} - \boldsymbol{\eta}, \tau) \tag{11}$$

where

$$\rho\psi_{\text{sink}} = \frac{Q}{4\pi r(1 - \nabla r_{ph} \boldsymbol{M})}$$

In the limit $\S \to 0$ and $\S Q = \boldsymbol{F}$ ($\boldsymbol{F}$ is the force vector), it is found that

$$p = -\nabla \cdot \left[\frac{F(\tau)}{4\pi r(1 - \nabla' r_{ph} \cdot \boldsymbol{M})}\right] \tag{12}$$

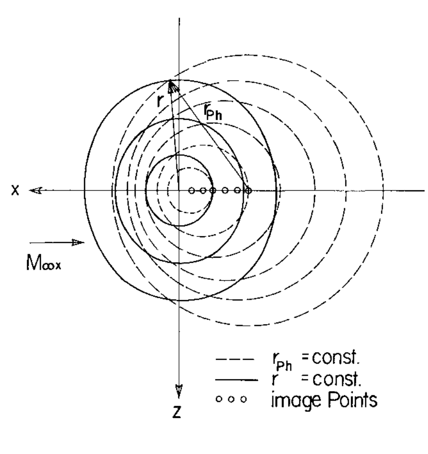


Fig. 1 Pressure monopole in a moving acoustic medium: r_{ph} = const are spheres of same phase, r = const are ellipsoidal surfaces of constant pressure amplitudes.

Thus, the pressure is given by the divergence of a vector field, and by using vector notation, the directions of the force $\boldsymbol{F}$ and the superimposed velocity $a\boldsymbol{M}$ are still arbitrary. Laws have to be established for the derivatives of $\tau = t - r_{ph}(\boldsymbol{x} - \boldsymbol{\eta}_o)$, $\boldsymbol{F}(\tau)$, and $\boldsymbol{M}(\tau)$. With $\boldsymbol{\eta}_o = \boldsymbol{\eta}_o(\tau)$, the derivation for which the retarded time τ is dependent on $\boldsymbol{x}$ has to be changed into the derivation ∇', for which the retarded time can be regarded as independent of $\boldsymbol{x}$. We then have

$$\nabla\tau = \frac{-\nabla' r_{ph}}{a(1 - \nabla' r_{ph} \cdot \boldsymbol{M})} \tag{13}$$

$$\nabla \cdot \boldsymbol{F}(\tau) = \frac{d\boldsymbol{F}}{d\tau} \cdot \nabla\tau$$

$$\nabla \cdot \boldsymbol{M}(\tau) = \frac{d\boldsymbol{M}}{d\tau} \cdot \nabla\tau \tag{14}$$

$$\nabla r_{ph} = \nabla' r_{ph} - \nabla' r_{ph} \cdot \frac{d\boldsymbol{\eta}_o}{d\tau} \nabla\tau$$

$$\nabla r = \nabla' r - \nabla' r \cdot \frac{d\boldsymbol{\eta}_o}{d\tau} \nabla\tau \tag{15}$$

Substituting Eqs. (13–15) into Eq. (12) finally gives, for the radiative terms,

$$P = \frac{1}{4\pi r a(1 - \nabla' r_{ph} \cdot \boldsymbol{M})^2} \times \left\{\nabla' r_{ph} \cdot \frac{d\boldsymbol{F}}{d\tau} + \frac{(\nabla' r_{ph} \cdot \boldsymbol{F})\left(\nabla' r_{ph} \cdot \frac{d\boldsymbol{M}}{d\tau}\right)}{(1 - \nabla' r_{ph} \cdot \boldsymbol{M})}\right\} \tag{16}$$

The near-field terms are found to be

$$P = \frac{1}{4\pi r^2(1 - \nabla' r_{ph} \cdot \boldsymbol{M})}\left\{\nabla' r \cdot \boldsymbol{F} + \frac{(\nabla' r_{ph} \cdot \boldsymbol{F})(\nabla' r \cdot \boldsymbol{M})}{(1 - \nabla' r_{ph} \cdot \boldsymbol{M})} - \frac{r\boldsymbol{F} \cdot \nabla'(\nabla' r_{ph} \cdot \boldsymbol{M})}{(1 - \nabla' r_{ph} \cdot \boldsymbol{M})} - \frac{r(\nabla' r_{ph} \cdot \boldsymbol{F})[\boldsymbol{M} \cdot \nabla'(\nabla' r_{ph} \cdot \boldsymbol{M})]}{(1 - \nabla' r_{ph} \cdot \boldsymbol{M})^2}\right\} \tag{17}$$

Equations (16) and (17) are a generalization of Lowson's[12] result for a still medium to the case of an uniformly moving medium. For $M_\infty = 0$, Eq. (16) coincides with Eq. (18) of Ref. 12. The vectors F, M, and $'r_{ph}$ are given in terms of the coordinates of the moving system. However, with the retarded time [see Eq. (2)], the signal is backdated to its time of generation. Also, Eq. (12), together with Eqs. (13–15), may be regarded as an extension of the dipole term in the Ffowcs-Williams-Hawkings equation.[3] Just as two equal and opposite dipoles at a very close distance result in a quadrupole, a force that is displaced parallel to itself by an acceleration, e.g., the propeller thrust, can also be regarded as a quadrupole-type noise source.[12] Thus, the first term on the right-hand side of Eq. (16) may be understood as a pressure doublet and the second term as a pressure quadrupole. Equation (16) implies linear wave propagation. For nonlinear propagation, different methods are recommended.[13]

If the magnitude of the force is constant and the uniform flow direction perpendicular to the propeller plane, then, Eqs. (16) and (17) coincide with the corresponding equation of Van de Vooren and Zandbergen,[6] who did not distinguish between the near and the far field. Due to the vector notation, the uniform flow direction is kept arbitrary in Eqs. (16) and (17). Thus, a propeller with angular inflow will be considered in the following section. Axial and in-plane forward speed are dealt with by this author in a separate report[14] and are inherent in the present formula as special cases. Also, the extension of Eq. (16) to cover the distribution of forces acting on the blade is a straightforward integration over the area of action.

Propeller with Angular Inflow

In practice, axial and in-plane components of forward speed occur very often simultaneously, e.g., a helicopter in forward flight and propeller aircraft at a high angle of attack during takeoff. If, for example, due to a change of the angle of attack of an aircraft, the propeller shaft is inclined by 1 deg with respect to the freestream flow, the angle of attack of the rotating blade will change by more than 1 deg during one revolution. This may result in considerable fluctuations for the thrust. From Fig. 2 it can be seen that the forward speed Mach number and its axial and in-plane components are given by

$$M_\infty^2 = M_{\infty x}^2 + M_{\infty z}^2 \tag{18}$$

and

$$\beta^2 = 1 - M_\infty^2, \quad \beta_x^2 = 1 - M_{\infty x}^2, \quad \beta_z^2 = 1 - M_{\infty z}^2 \tag{19}$$

It should be noted that in order to be consistent with the mathematical notation, the components of the freestream velocity aM_∞ are chosen to be in the negative x and z direction. Thus (see Fig. 2), the positive z direction is in the downward direction. Fundamental solutions can be found from a rotation of the coordinate system (shown in Fig. 2) such that one axis is in the negative direction of the uniform flow. Lorentz-Galilean transformation[6] may be applied and with the force acting on the blade at a radius R from the hub, and at angular position θ we have for the phase radius

$$r_{ph} = [M_{\infty x}x + M_{\infty z}(z - R\sin\theta) + r]/\beta^2 \tag{20}$$

where

$$r^2 = \beta_z^2 x^2 + \beta_x^2 (z - R\sin\theta)^2 + 2M_{\infty x}M_{\infty z}(z - R\sin\theta)x + \beta^2(y - R\cos\theta)^2 \tag{21}$$

and

$$\theta = \Omega\tau \tag{22}$$

with Ω as angular velocity of the blade ($\theta = 0$ along the positive y axis). With r_{ph} taken from Eq. (20), Eq. (2) can be used for the retarded time. The convection Mach number $\boldsymbol{M}$ of the force is the Mach number $\boldsymbol{M_R}$ in the direction of rotation and may be written as

$$\boldsymbol{M_R} = (O, -M_R\sin\theta, M_R\cos\theta) \tag{23}$$

and

$$M_R = \Omega R/a \tag{24}$$

The force per blade area element fluctuates in magnitude with θ. The force magnitude F may be represented by Fourier azimuth components so that

$$F = \sum_{s=0}^{s=\infty} F_s \cos(s|\theta + \zeta|) \tag{25}$$

where ζ determines the phase for $\theta = 0$. The order of the blade-loading harmonic is designated by s. The amplitude F_x may be written in terms of steady blade loading as

$$F_s = F_o\alpha_s \tag{26}$$

where α_s is the harmonic blade-loading coefficient and F_o the magnitude of the steady blade force given by average values of blade-element thrust and torque force, i.e., by T_o and D_o. The forces on the air are equal and opposite to that on the blade. Therefore, it is a thrust in the negative x direction and a torque force in the direction of rotation:

$$\boldsymbol{F} = \left(-T_o\frac{F}{F_o}, \quad -D_o\frac{F}{F_o}\sin\theta, \quad D_o\frac{F}{F_o}\cos\theta\right) \tag{27}$$

Because of the angular inflow, the directionality of the sound pressure amplitudes is not axisymmetric with respect to the shaft axis (x axis). Therefore, the observation cannot be restricted to the y-x plane. For the relationship between observer and source, the angle δ_o (see Fig. 3) is introduced:

$$\cos\delta_o = (y - R\cos\theta)/r_{yz}$$
$$\sin\delta_o = [M_{\infty z}(M_{\infty x}x + r) + \beta_x^2(z - R\sin\theta)]/(\beta^2 r_{yz}) \tag{28}$$

where

$$r_{yz}^2 = (y - R\cos\theta)^2 + [M_{\infty z}(M_{\infty x}x + r) + \beta_x^2(z - R\sin\theta)]^2/\beta^4 \tag{29}$$

It should be noted that, for example, the quantity r or its far-field approximation r_1 [Eq. (34)] are not simply the geometric distances between observer and rotating source or between observer and the hub of the origin (see Fig. 3). From Eq. (34) it follows that r_1 describes the surface of an ellipsoid. This is because in a moving coordinate system, the pressure amplitude due to a pressure monopole is constant on the surface of an ellipsoid (see Fig. 1). Only for the case of zero forward speed r and r_1 are reduced to the geometric distances shown in Fig. 3. Similar considerations apply for the definitions of δ_o, δ_1 [Eq. (37)], r_{yz}, r_{yz} [Eq. (35)], r_x [Eq. (30)] and r_{x1} [Eq. (36)]. The introduction of r_x, given as

$$r_x = \{M_{\infty x}[M_{\infty x}(z - R\sin\theta) + r] + \beta_z^2 x\}/\beta^2 \tag{30}$$

and the substitution of Eqs. (18–30) into Eq. (16) gives

$$p = \frac{-\alpha_s\Omega}{4\pi ar\left[1 + M_R\dfrac{r_{y,z}}{r}\sin(\theta - \delta_o)\right]^2} \cdot \Bigg\{\left[T_o\frac{r_x}{r} + \frac{D_o r_{y,z}}{r}\sin(\theta - \delta_o)\right](-s)\sin(s|\theta + \zeta|)$$
$$+ D_o\frac{r_{y,z}\cos(\theta - \delta_o) - R}{r}\cos(s|\theta + \zeta|)$$
$$- \left[T_o\frac{r_x}{r} + D_o\frac{r_{y,z}}{r}\sin(\theta - \delta_o)\right]$$
$$\times (r_{y,z}\cos(\theta - \delta_o) - R)M_R$$
$$\times \cos(s|\theta + \zeta|)\Bigg/\left[1 + M_R\frac{r_{y,z}}{r}\sin(\theta - \delta_o)\right]\Bigg\} \tag{31}$$

In the far field $r \gg R$:

$$r_{ph} \sim r_{ph1} - \frac{r_{yz1}}{r1}R\cos(\theta - \delta_1) \tag{32}$$

where

$$r_{ph1} = (M_{\infty x}x + M_{\infty z}z + r_1)/\beta^2 \tag{33}$$

$$r_1 = \sqrt{\beta_z^2x^2 + \beta_x^2z^2 + \beta^2y^2 + 2xzM_{\infty x}M_{\infty z}} \tag{34}$$

$$r_{yz1}^2 = y^2 + [M_{\infty z}(M_{\infty x}x + r_1) + \beta_x^2 z]^2/\beta^4 \tag{35}$$

$$r_{x1} = [M_{\infty x}(M_{\infty z} + r_1) + \beta_z^2 x]/\beta^2 \tag{36}$$

and δ_1 from

$$\cos\delta_1 = \frac{y}{r_{yz1}}, \qquad \sin\delta_1 = [M_{\infty z}(M_{\infty x}x + r_1) + \beta_x^2 z]/\beta^2 r_{yz1} \tag{37}$$

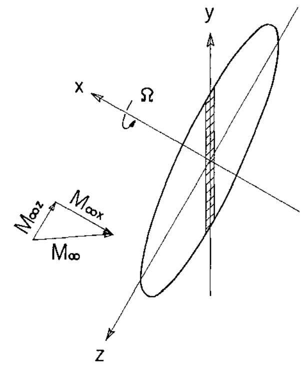


Fig. 2 Coordinate system for a propeller with angular inflow.

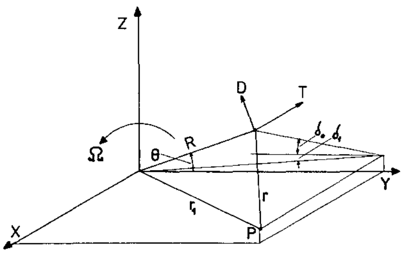


Fig. 3 Rotating thrust *T* and drag *D* with the observer at point *P*.

Using these approximations, a Fourier analysis for the far-field frequency spectrum can be made. The sound pressure radiation then follows from an integration of Eq. (31) over ψ, where

$$\psi = \Omega(t - r_{ph1}/a) - \delta_1 = (\theta - \delta_1) - M_R \frac{p_1}{r_1}\cos(\theta - \delta_1) \quad (38)$$

Changing the variable of integration from ψ to $\theta - \delta_1$ and integration by parts finally gives in complex notation

$$P_{s,mB} = \frac{-imB\Omega\alpha_s}{4\pi a r_1}\exp(imB\psi) \cdot \frac{1}{\pi}\int_o^{2\pi}\left\{\left[T_o\frac{r_{x1}}{r_1} + D_o\frac{r_{yz1}}{r_1}\sin(\theta-\delta_1)\right]\right. \cdot \frac{1}{2}\exp\left\{i\left[mBM_R\frac{r_{yz1}}{r_1}\cos(\theta-\delta_1) - (mB+s)(\theta-\delta_1) - s(\delta_1+\zeta)\right]\right\} \left. + \exp\left\{i\left[mBM_R\frac{r_{yz1}}{r_1}\cos(\theta-\delta_1) - (mB+s)(\theta-\delta_1) - s(\delta_1+\zeta)\right]\right\}\right\} \mathrm{d}(\theta-\delta_1) \quad (39)$$

The subscript *s,mB* means sound pressure harmonic *mB* due to the blade-loading harmonic *s*. With $q - mB - s$ and $q_+ = Mb + s$, the Fourier coefficients of the sound pressure are given as

$$SP_{s,mB} = \frac{mB\alpha_s\Omega}{4\pi a r_1} \cdot \left\{\left(T_o\frac{r_{x1}}{r_1} - D_o\frac{q_-}{mBM_R}\right)J_{q_-}\left(mBM_R\frac{r_{yz1}}{r_1}\right) + \left(T_o\frac{r_{x1}}{r_1} - D_o\frac{q_+}{mBM_R}\right)J_{q_+}\left(mBM_R\frac{r_{yz1}}{r_1}\right)\right\} \quad (40)$$

where $SP_{s,mB}$ means sound pressure amplitude of the *mB*th sound harmonic generated by the *s* loading harmonic. The preceding equation describes the effects of forward speed and asymmetric disk loading on the Fourier coefficients of the different sound harmonics and passing to the geometric far field is the only approximation leading to Eq. (40). If $M_{\infty z} = 0$ and $s = 0$, which is the case for steady disk loading, then $q = mB$, $\alpha_s = 1$, and Eq. (40) reduces to the formula of Garrick and Watkins.[4] If, in addition, the freestream Mach number is zero and ($M_{\infty x} = 0$), the formulas of Gutin[16] and Lowson[12] are reproduced, and for $s \neq 0$ and $M_{\infty x} = M_{\infty z} = 0$, Wright's[8] formula resulted.

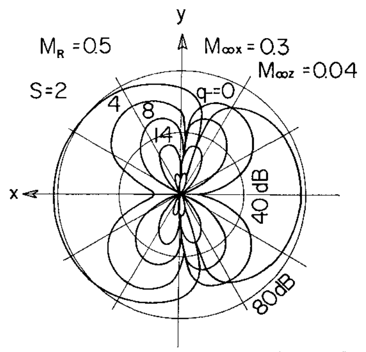
From Eqs. (34), (36), and (40), it can be seen that the sound pressure amplitude due to thrust is proportional to $(\beta_z^2 \pm M_{\infty z})/\beta^2$ for radiation into the *x* direction and to $M_{\infty x}$ $(\beta_x \pm M_{\infty z})/(\beta^2\beta_x)$ for radiation into the *z* direction, where the plus signs apply for the positive directions. Following Wright's[17] paper, a mode Mach number of open-mode propagation may be defined as the ratio of the Bessel function argument and order. The Bessel function argument is also affected by the forward speed. The radiation efficiency is governed by the Bessel function argument. This is for radiation into

1) the *x* direction	$mBM_RM_{\infty z}(\beta_z \pm M_{\infty x})/(\beta^2\beta_z)$
2) the *y* direction	$mBM_R\beta_x/\beta^2$
3) the *z* direction	$mBM_R(\beta_x \pm M_{\infty z})/\beta^2$

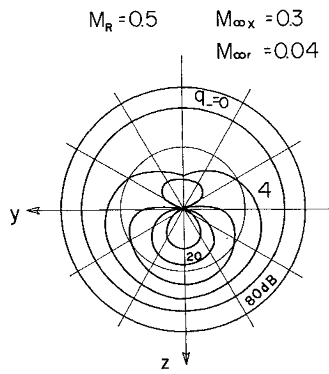
Line 2 gives the argument found in many papers for axial inflow problems. The results are not changed to the side (see Fig. 4b). However, line 3 shows that the argument is increased below and decreased above the propeller (see Fig. 4c).

Division of the given Bessel function arguments by q_- or q_+ gives the desired mode Mach number. For $q_- = 0$, the mode Mach number is infinite. The $q_- = 0$ mode is rotating at an infinite speed, and the polar diagram will be wholly thrust-derived (see Fig. 4). The sound power radiated into the discrete frequency spectrum has a maximum about $q_- = 0$. If the mode Mach number is $\sqrt{2}$, the sound wave will make an angle of 45 deg with the circular path of the force. For high, supersonic-mode Mach numbers, the sound power output may be significant, whereas for subsonic-mode Mach numbers, the power output may be negligible. Thus, prior to any numerical evaluation, information is available on the wave mode geometry and intensity. Since the blade-tip Mach number is subsonic, the introduction of the mode Mach number demonstrates the analogy between unsteady subsonic flow and steady supersonic flow.

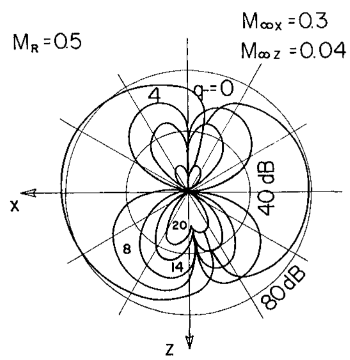
The most accurate way to evaluate the preceding formulas probably would be to use free-flight test data for the distribution of the unsteady pressure all over the blade and to extend Eq. (16) to cover the distribution over the blade. However, the acquisition of reliable data by free-flight measurements, wind-tunnel measurements, or some theory is a research program in itself, and such data were not available to the author. Therefore, α_s coefficients are taken from Wright's[8] example for an S58 helicopter in forward flight. In addition, it is assumed that the forces acting on one blade can be replaced by an effective force per blade rotating at 80% of the propeller radius.[15] A large number of polar diagrams illustrating the three-dimensional directionality of the sound pressure amplitudes for the different sound harmonics (*mB*) and the different blade loadings (α_s) could be received from Eq. (40). In order to demonstrate the effect of in-plane components of forward speed on the directionality the second-order blade loading, ($s = 2$, $\alpha_2 = 0.1496$) is taken. A blade loading with an order higher than $s = 2$ would result in a directionality with many loops and thus deliver a more complicated picture than for $s = 2$. For the same reason, only the modes for $q_- = 0, 4, 8, 14,$ and 20 were selected. The q_+ modes were found to be insignificant. The resulting polar diagram of Fig. 4 is three-dimensional. For



a) x, y plane orthogonal to the propeller disk and parallel to the uniform flow



b) Propeller disk plane (y, z)



c) x, z plane

Fig. 4 Three-dimensional polar diagrams for discrete frequency due to a propeller in sideslip. Blade loading harmonic $s = 2$, $\alpha_s = 0.1496$, $M_R = 0.5$, $B = 2$, $T_{BL} = 200$ kp, $D_{BL} = 20$ kp, 2400 rev/min distance = 100 m from the hub. A ratio of $T_o/D_o = 10$ is used.

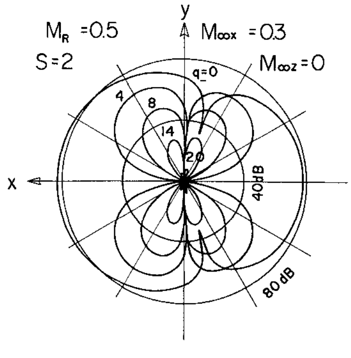


Fig. 5 Axisymmetric polar diagram without angular inflow ($M_{\infty z} = 0$), all other parameters are same as in Fig. 4.

comparison, the axisymmetric directionality for the same blade loading harmonic without angular inflow ($M_{\infty z} = 0$) is shown in Fig. 5. Except for the $q_- = 0$ mode, the differences in the directionality are significant. Due to angular inflow, there is an amplification of the sound pressure in the forward and downward direction, i.e., in the positive x and z direction, as shown in Figs. 4b and 4c. Since the in-plane component of forward speed $aM_{\infty z}$ is chosen to be in the negative z direction (see Fig. 2), the z axis is positive in the downward direction. The directionality of the sound pressure amplitude is symmetric with respect to the z axis and the x axis because of the symmetry of the flow (see Figs. 4a and 4b). Thus, angular inflow affects the sound generation and propagation, and these effects can be taken into account via the blade-loading harmonics and the three-dimensional directionality.

Final Consideration

Helicopter rotors and aircraft propellers in takeoff or landing generally are operating with inflow components of forward speed. To account for the effects of angular inflow on the sound generation and progagation, a basic solution for the sound pressure is first derived [Eq. (16)]. This solution is given in vector notation and can be applied for the case of a point force with an arbitrary velocity superimposed on a uniform flow. The uniform flow direction is also arbitrary. The propeller blade forces are represented by a point force acting at an effective radius. The force vectors and velocity vectors are introduced into the basic equation, and the sound pressure amplitudes for a propeller with angular inflow are obtained by a conventional Fourier series expansion. The polar diagrams for the directionality of the sound pressure amplitudes show that the effect of angular inflow is significant. The final formula for the moving system, Eq. (40), is simple and fairly easy to apply. Formulas for the sound pressure in an airfixed system can be obtained by applying an inverse Galilean transformation to the given formulas.

At transonic blade-tip speeds, thickness noise becomes the most important noise. For this case, different methods have to be applied.[10,18]

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